Chapter 3

Electrodynamic Instruments

Objectives

- To understand the basic construction of a dynamometer instrument.
- Explain basic operation and development of torque expressions for ammeter, voltmeter and wattmeter.
- Study of ammeter, voltmeter and wattmeter connections.
- To investigate the errors involve in wattmeter readings and its compensation.
- Understanding the effect of inductance of voltage coil (moving coil) on the wattmeter readings.

3.1 Introduction

Electrodynamic type instruments are similar to the PMMC-type elements except that the magnet is replaced by two serially connected fixed coils that produce the magnetic field when energized (see Fig. 3-1). The fixed coils are spaced far enough apart to allow passage of the shaft of the movable coil. The movable coil carries a pointer, which is balanced by counter weights. Its rotation is controlled by springs. The motor torque is proportional to the product of the currents in the moving and fixed coils. If the current is reversed, the field polarity and the polarity of the moving coil reverse at the same time, and the turning force continues in the original direction.

Electrodynamometer-type meters

The electrodynamometer-type meter differs from the galvanometer types we have just studied in that two fixed coils are used to produce the magnetic field instead of a permanent magnet. Two movable coils are also used in the electrodynamometer meter. The electrodynamometer meter is most commonly found in various types of power meters.

Since the reversing the current direction does not reverse the turning force, this type of instruments can be used to measure AC or DC current, voltage, or its major application as a wattmeter for power measurement. In the first two cases, the moving and fixed are serially connected. For power measurement, one of the coils (usually the fixed coils) passes the load current and other coil passes a current proportional to the load voltage. Air friction damping is employed for these instruments and is provided by a pair of Aluminum-vanes attached to the spindle at the bottom. These vanes move in a sector shaped chamber. Cost and performance compared with the other types of instruments restrict the use of this design to AC or DC power measurement. Electro-dynamic meters are typically expensive but have the advantage of being more accurate than moving coil and moving iron instrument but its sensitivity is low. Similar to moving iron vane instruments, the electro dynamic instruments are true RMS responding meters. When electro dynamic instruments used for power measurement its scale is linear because it predicts the average power delivered to the load and it is calibrated in average values for AC.

Voltage, current and power can all be measured if the fixed and moving coils are connected appropriately. Other parts of the instruments are described briefly below:

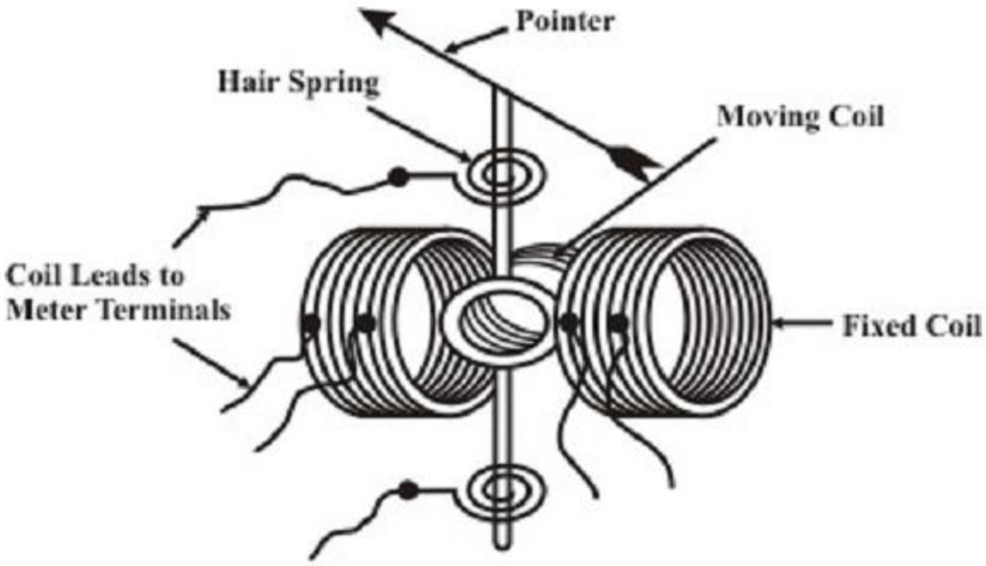


Figure 3-1 Electro dynamic (or Dynamometer) type Instruments:

Fixed coil: The magnetic field is produced by the fixed coil which is divided into two sections to give more uniform field near the centre and to allow passage of the instrument shaft.

Moving coil: The moving coil is wound either as a self-sustaining coil or else on a non-magnetic former. A metallic former cannot be used, as eddy currents would be induced in it by alternating field. Light but rigid construction is used for the moving coil. It should be noted that both fixed and moving coils are air cored.

Springs: The controlling torque is provided by two control springs. These hairsprings also act as leads of current to the moving coil.

Dampers: Air friction damping is employed for these instruments and is provided by a pair of Aluminum-vanes attached to the spindle at the bottom. These vanes move in a sector shaped chamber.

Shielding: Since the magnetic field produced by fixed coils is weaker than that in other types of instruments, these meters need a special magnetic shielding. Electro-dynamic instruments are effectively shielded from the effects of external magnetic fields by enclosing the mechanism in a laminated iron hollow cylinder with closed ends.

As shown in Figure 3-2, the fixed coils are connected in series and positioned coaxially (in line) with a space between them. The two movable coils are also positioned coaxially and are connected in series. The two pairs of coils (fixed pair and movable pair) are also connected in series with each other.

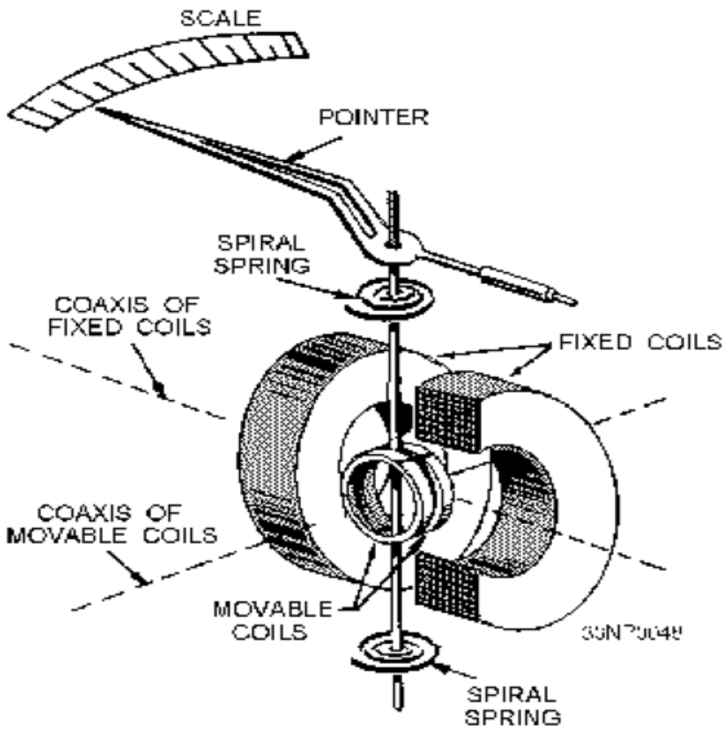


Figure 3-2.—Internal construction of an electrodynamometer.

The movable coil is pivot-mounted between the fixed coils. The main shaft on which the movable coils are mounted is restrained by spiral springs that restore the pointer to zero when no current is flowing through the coil. These springs also act as conductors for delivering current to the movable coils. Since these conducting springs are very small, the meter cannot carry a high value of current.

Meter accuracy

The meter is mechanically damped by means of aluminum vanes that move in enclosed air chambers. Although very accurate, electrodynamometer-type meters do not have the sensitivity of the D’Arsonval-type meter movement. For this reason, you will not find them used outside of the laboratory environment to a large extent.

Meter movement

The primary advantage of the electrodynamometer-type meter movement is that it can be used to measure alternating as well as direct current. If you apply alternating current to the standard galvanometer-type meter, it will not produce a usable reading. Instead, the meter will vibrate at or near the zero reading. On one-half cycle of the ac, the meter is deflected to the left and on the other half cycle to the right. Since the frequencies you will be measuring are 60 hertz or greater, the meter is incapable of mechanically responding at this speed. The result is simply a vibration near the zero point; in addition, no useful reading of voltage or current is obtained. This problem does not exist with the electrodynamometer-type movement. Current flow through the stationary (fixed) coils sets up a magnetic field. Current flow through the moving coils sets up an opposing magnetic field. With two magnetic fields opposing, the pointer deflects to the right. If the current reverses direction, the magnetic fields of both sets of coils will be reversed. With both fields reversed, the coils still oppose each other, and the pointer still deflects to the right. Therefore, no rectifying devices are required to enable the electrodynamometer meter movement to read both ac and dc. Rectifying devices are required for the D'Arsonval-type meter movement to enable it to be used for measuring ac voltages and currents.

3.2 Operating Principle

Let us consider the currents in the fixed and moving coils are if and im respectively. The action of electrodynamic instrument depends upon the force exerted between fixed and moving coils carrying current. The flux density B (wb/m2) produced by the fixed coil is proportional to if (fixed coil current).

The force on the conductors of the moving coil, for a given strength field, will proportional to im (moving coil current) and the number of turns ' N' of the moving coil. In case of ammeter and voltmeter fixed and moving coils are connected in series and the developed torque is due to the interaction of the magnetic fields produced by currents in the fixed and moving coils and thus it will be proportional to:

$$i^2 \; (i_f = i_m = i).$$

Thus, dynamic instruments can be used for dc and ac measurements.

Expression for developed torque:

Let us assume that the fixed and moving coils having self-inductances Lf and Lm respectively. Further it is assumed that the mutual inductance between the fixed and movable coils is M

Total energy stored in the magnetic field of the coils is given by

$$W = \frac{1}{2} L_f i_f^2 + \frac{1}{2} L_m i_m^2 + M i_f i_m \qquad (3\text{-}1)$$

where if and im are the currents through the fixed and moving coils. From equation (43.1) one can write the expression for torque developed as

$$T_d = \frac{dW}{d\theta} = i_f i_m \frac{dM}{d\theta} \qquad (3\text{-}2)$$

Note Lf and Lm are not functions of θ but the mutual inductance 'M ' between the coils is a function of the deflection θ (i.e relative position of moving coil). The equivalent inductance between fixed and moving coils can be found out as $L_{eq}=L_f+L_m+2M$ (cumulative manner) and from this one can find the mutual inductance between them as

$$M = \frac{1}{2}\left[L_{eq} - (L_f + L_m) \right] \qquad (3\text{-}3)$$

With all deflection type instruments, however, the mutual inductance varies with the relative positions of the moving and fixed coils. The maximum value Mmax of the mutual inductance occurs when the axes of the moving and fixed coils are aligned with θ = 180º, as this position gives the maximum flux linkage between coils. When θ = 0º, M= - M max. If the plane of the moving coil is at an angle θ with the direction of B that produced by the fixed coil, then the mutual inductance M is expressed by

$$M = -\ M_{max} \cos \theta \qquad (3\text{-}4)$$

D.C operation: Expression (43.2) for the developed torque is rewritten by setting ($i_f = I_f (d.c)$) im=Im(dc)

$$T_d = I_f I_m \frac{dM}{d\theta} = I_f I_m M_{max} \sin\theta \qquad (3\text{-}5)$$

If the control is due to spiral springs, the controlling torque is proportional to the angle of deflection θ.

Controlling torque

$$T_c = k_s\,\theta \qquad (3\text{-}6)$$

where k_s is the spring constant.

Note that Td = Tc at steady deflection, i.e,

$$I_f I_m \frac{dM}{d\theta} = k_s \theta$$

$$\therefore \qquad \theta = \frac{I_f I_m}{k_s}\left(\frac{dM}{d\theta}\right) \qquad (3\text{-}7)$$

A.C operation: The dynamometer instrument is used to measure alternating current or voltage, the moving coil–due to its inertia takes up a position where the average deflecting torque over a complete cycle is balanced by the restoring torque of the spiral spring. The deflecting torque is proportional to the mean value of the square of the current or voltage (note both coils are connected in series for ammeters or voltmeters), and the instrument scale can therefore be calibrated to read r.m.s values of alternating current or voltage.

Average deflecting torque

$$T_{d,av} = \frac{1}{T}\int_0^T i_f(t)\, i_m(t)\,\frac{dM}{d\theta}\,dt$$

Let $i_f(t) = I_{max,f}\sin\omega t$, $i_m(t) = I_{max.m}\sin(\omega t - \alpha)$ where 'α' is the phase–angle between two currents

$$T_{d,\,av} = I_1\, I_2 \cos\alpha \frac{dM}{d\theta} \qquad (3\text{-}8)$$

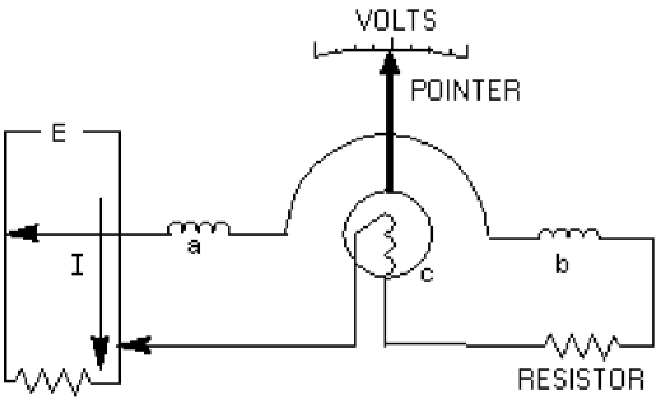
The traditional analog wattmeter is an electrodynamic instrument. The device consists of a pair of fixed coils, known as current coils, and a movable coil known as the potential coil.

The current coils connected in series with the circuit, while the potential coil is connected in parallel. Also, on analog wattmeters, the potential coil carries a needle that moves over a scale to indicate the measurement. A current flowing through the current coil generates an electromagnetic field around the coil. The strength of this field is proportional to the line current and in phase with it. The potential coil has, as a general rule, a high-value resistor connected in series with it to reduce the current that flows through it.

The result of this arrangement is that on a dc circuit, the deflection of the needle is proportional to both the current and the voltage, thus conforming to the equation W=VA or P=VI.

3.3 Electrodynamic Voltmeter

When an electrodynamometer is used as a voltmeter, no problems in construction are encountered because the current required is not more than 0.1 ampere. This amount of current can be handled easily by the spiral springs. When the electrodynamometer is used as a voltmeter, its internal connections and construction are as shown in view A of figure 3-3. Fixed coils ***a*** and ***b*** are wound of fine wire since the current flow through them will not exceed 0.1 ampere. They are connected directly in series with movable coil c and the series current-limiting resistor.



(A) VOLTMETER

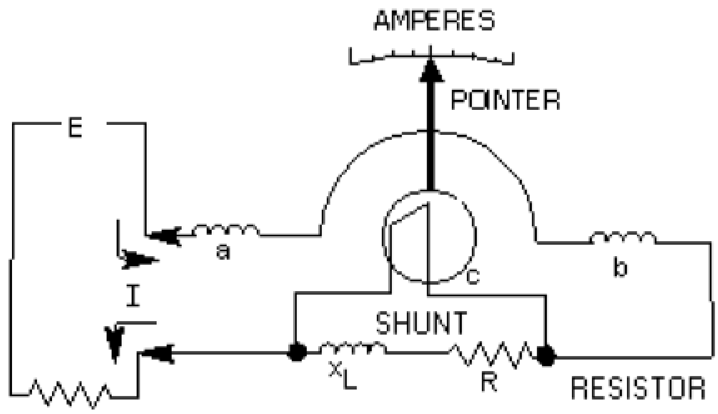


Figure 3-3.—Circuit arrangement of electrodynamometer for use as a voltmeter and an ammeter.

3.4 Electrodynamic Ammeter

When the electrodynamometer is used as an ammeter, a special type of construction must be used. This is because the large currents that flow through the meter cannot be carried through the moving coils. In the ammeter in view B of Fig. 3-3, stationary coils a and b are wound of heavier wire to carry up to 5.0 amperes. An inductive shunt (XL) is wired in parallel with the moving coils and permits only a small part of the total current to flow through the moving coil. The current flowing through the moving coil is directly proportional to the total current flowing through the instrument. The shunt has the same ratio of reactance to resistance as the moving coil does. Therefore, the instrument will be reasonably correct at frequencies at which it is used if ac currents are to be measured.

3.5 Electrodynamic Wattmeter

Electric power is measured by means of a wattmeter. This instrument is of the electrodynamometer type. As shown in figure 3-4, it consists of a pair of fixed coils, known as current coils, and a moving coil, called the voltage (potential) coil. The fixed current coils are wound with a few turns of a relatively large conductor. The voltage coil is wound with many turns of fine wire. It is mounted on a shaft that is supported in jeweled bearings so that it can turn inside the stationary coils. The movable coil carries a needle (pointer) that moves over a suitably graduated scale. Coil springs hold the needle at the zero position in the absence of a signal.

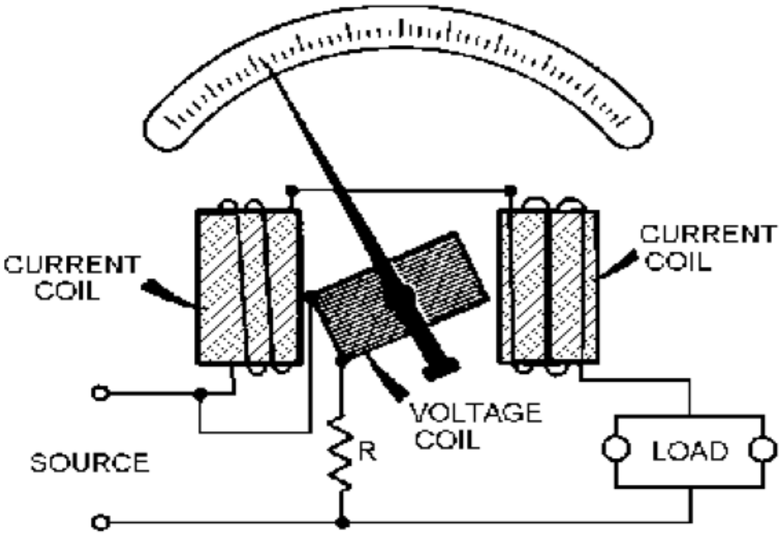


Figure 3-4.—Simplified electrodynamometer wattmeter circuit.

3.5.1 Wattmeter Connection

The current coil of the wattmeter is connected in series with the circuit (load), and the voltage coil is connected across the line. When line current flows through the current coil of a wattmeter, a field is set up around the coil. The strength of this field is in phase with and proportional to the line current. The voltage coil of the wattmeter generally has a high-resistance resistor connected in series with it. The purpose for this connection is to make the voltage-coil circuit of the meter as purely resistive as possible. As a result, current in the voltage circuit is practically in phase with line voltage. Therefore, when voltage is impressed on the voltage circuit, current is proportional to and in phase with the line voltage. Figure 3-5 shows the proper way to connect a wattmeter into a circuit.

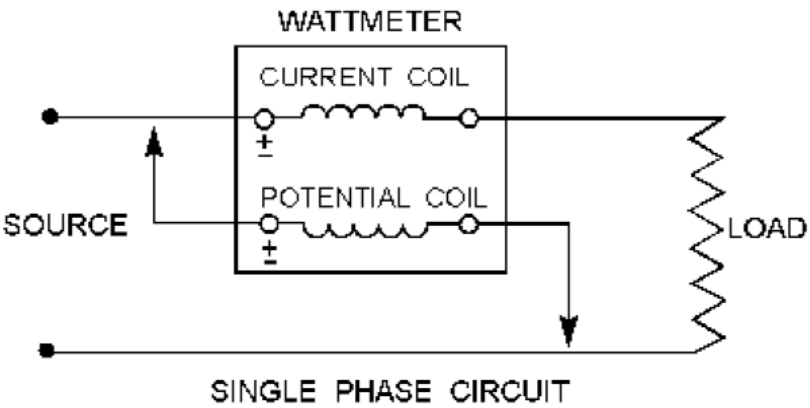


Figure 3-5.—Wattmeter connection.

3.5.2 Wattmeter Errors

Electrodynamic wattmeters are subject to errors arising from such factors as temperature and frequency. For example, heat through the coils eventually causes the small springs attached to the pointer to lengthen and lose tension, which produces deflection errors. Large currents through the wattmeter also produce a noticeable deflection error. These errors are caused by the heat (I2R) loss through coils from the application of high currents. Because of this, the maximum current range of electrodynamic wattmeters is normally restricted to approximately 20 amperes. The voltage range of wattmeters is usually limited to several hundred volts because of heat dissipation within the voltage circuit. However, the voltage range can be extended by the use of voltage multipliers. Good-quality, portable wattmeters usually have an accuracy of 0.2 to 0.25 percent. You must remember, though, that electrodynamic wattmeter errors increase with frequency. For the higher frequency and power ranges, special types of wattmeters are made specifically for those ranges. We will discuss two such wattmeters in chapter 5 of this module.

3.5.3 Wattmeter Overloads

The wattmeter consists of two circuits, either of which will be damaged if too much current passes through them. You should be especially aware of this fact because the reading on the instrument will not tell you whether or not the coils are being overheated. If an ammeter or voltmeter is overloaded, the pointer will indicate beyond the upper limit of its scale. In the wattmeter, both the current and potential circuit may carry such an overload that their insulations burn; yet the pointer may be only part of the way up the scale. This is because the position of the pointer depends upon the power factor of the circuit as well as upon the voltage and current. Therefore, a low power-factor circuit will provide a very low reading on the wattmeter. The reading will be low, even when the current and voltage circuits are loaded to the maximum safe limit. The safe rating for each wattmeter is always distinctly rated, not in watts, but in volts and amperes.

The electrodynamic wattmeter is used to measure power taken from ac or dc power sources. The electrodynamic wattmeter, shown in figure 3-10, uses the reaction between the magnetic fields of two current-carrying coils (or sets of coils), one fixed and the other movable. When the current through the fixed-position field winding(s) is the same as current through the load and the current through the moving coil is proportional to the load voltage, then the instantaneous pointer deflection is proportional to the instantaneous power. Since the moving pointer cannot follow the rapid variations in torque because of its momentum, it assumes a deflection proportional to the average power. The dynamometer-type wattmeter automatically compensates for the power factor error of the circuit under test. It indicates only the instantaneous power resulting from in-phase values of current and voltage. With out-of-phase relationships, a current peak through the moving coil never occurs at the same instant as the voltage peak across the load, resulting in less pointer deflection than when the current and voltage are in phase. The simple meter shown in figure 3-10 is not compensated. When the load is disconnected, this meter will still indicate that power is being consumed in the circuit. This difficulty can be eliminated by incorporating two compensating windings, mounted with the primary fixed-coil current windings, as shown in figure 3-11. These stationary windings are used to produce a magnetic flux proportional to the current through the movable coil. As shown by the arrows, the currents through the primary movable coil and the compensating coil flow in opposite directions, producing a torque caused by the opposing magnetic fields. These opposing fields cancel. Hence, with the load removed from the circuit, the meter will indicate zero power through the load.

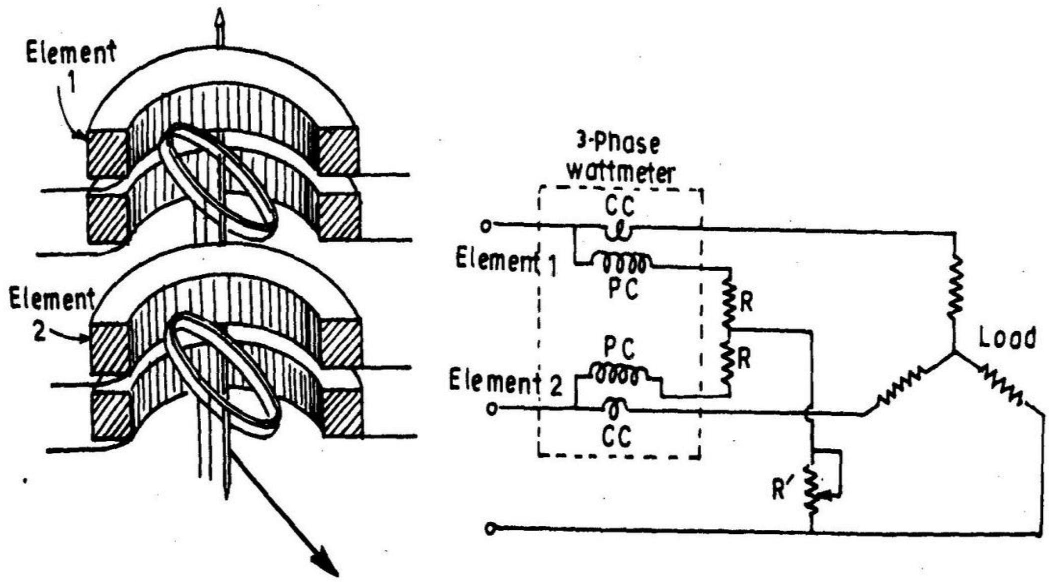


Figure 3-10 The electrodynamic wattmeter

Figure 3-11.

3.6 Dynamometer Type Moving Coil Ammeter and Voltmeter

As said earlier, in dynamometer type ammeters and voltmeters air gap flux is produced by an electromagnet, in the form of one or two fixed coils. The fixed coils carry the current proportional to the current or voltage to be measured and are connected in series or parallel with the moving coil. The coils, in general, are air cored in order to avoid errors due to hysteresis, eddy current and other errors when the meter is used for a.c measurement. Since, the flux in dynamometer type is only about 3 to 4 percent of that in permanent magnet type, the number of ampere-turns on the moving coil, required to produce same torque, is large. This can be achieved by increasing the number of turns in the moving coil. But the increase of number of turns increases the weight and inertia of the moving system. Dynamometer type instruments can be used both in d.c and a.c measurements.

Generally used dynamometer type instrument is wattmeter. Its use as ammeter and voltmeter is limited. But because of its excellent accuracy, dynamometer type ammeters and voltmeters, calibrated in d.c circuits, are used as substandard meters for checking and calibrating a.c instruments. Through this type of instruments operate both on d.c and a.c circuits, they are used for precise measurements on a.c circuits and not on d.c circuits as these instruments compare unfavorable with the permanent magnet instruments because of the following disadvantages:

1. Weight / torque ration is high
2. Because of weak operating field these instruments are very sensitive to stray field
3. Because of heavy moving system the frictional error is more. The internal heating is also more
4. The scale is not uniform
5. These instruments are costlier than the permanent magnet instruments

Operating principle

The operating torque is produced by the reaction between the magnetic field of the fixed coils and the current through the moving coil. The torque produced remains always positive regardless of the direction of current. This happens because the fact that with the change of direction of the current in the moving coil the field of the fixed coils also changes their directions. To derive the torque equation, consider the energy stored in the magnetic circuits. The total stored magnetic energy is given by

$$W = \frac{i_1^2 L_1}{2} + \frac{i_2^2 L_2}{2} + i_1 i_2 M \qquad (3\text{-}9)$$

where i_1 and i_2 are instantaneous values of two coil currents.
If the moving system rotates through a small angle $d\theta$ and the corresponding change in stored magnetic energy is dW, the work done in moving system

$$T_d \, d\theta = dW$$

$$\therefore \qquad T_d = \frac{dW}{d\theta}$$

From (3-9), the rate of change of energy,

$$\frac{dW}{d\theta} = i_1 i_2 \frac{dM}{d\theta}$$

Since, self inductances L1 and L2 are independent of θ . Hence,

$$T_d = i_1 i_2 \frac{dM}{d\theta} \qquad (3\text{-}10)$$

In case of direct current

$$T_D = I_1 I_2 \frac{dM}{d\theta} \qquad (3\text{-}11)$$

When the fixed coils and the moving coil are in series and hence carry the same current,

$$T_D = I^2 \frac{dM}{d\theta} \qquad (3\text{-}12)$$

In case of the steady state sinusoidal current,

$$i_1 = \sqrt{2} I_1 \sin \omega t$$

$$i_2 = \sqrt{2} I_2 \sin (\omega t - \phi) \qquad (3\text{-}13)$$

where I_1 and I_2 are the r.m.s value of two coil currents, and ϕ is the phase angle between I_1 and I_2. Hence,

$$T_d = 2 I_1 I_2 \sin \omega t \sin(\omega t - \phi) \frac{dM}{d\theta} \quad (3\text{-}14)$$

and the average torque,

$$T_d = \frac{1}{T} \int_0^T 2 I_1 I_2 \frac{dm}{d\theta} \sin \omega t \sin(\omega t - \phi)\, dt$$

$$= I_1 I_2 \frac{dM}{d\theta} \cos \phi \quad (3\text{-}15)$$

where T is the time period of the sinusoidal current.

When the fixed coils and the moving coil are in series, they carry same current. Hence, ϕ=0 and $I_1 = I_2$=I, and the deflecting torque,

$$T_D = I^2 \frac{dM}{d\theta} \quad (3\text{-}16)$$

In practice, the ratio dM/dθ is made constant over the working range by suitable design of sections and radii of coils.

Connection of ammeters

For low range ammeters (about 200 mA) the moving and fixed coils are connected in series as shown in Fig. 3.12(a). The current carrying capacity of the control spring limits the range of the ammeters. Ammeters with range higher than 200 mA are used with a shunt as shown in Fig. 3.12(b).

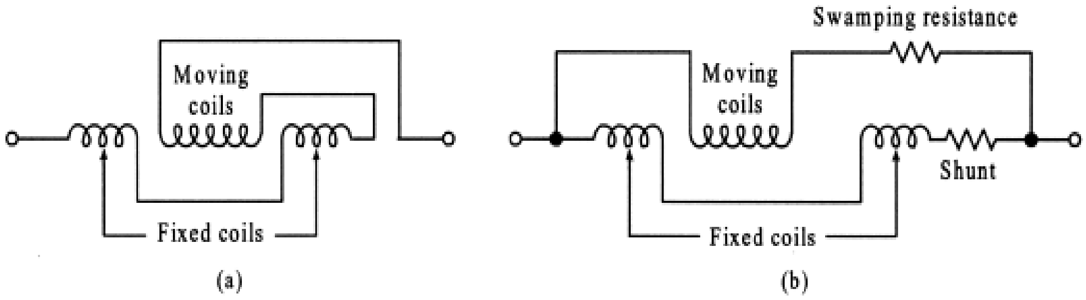


Fig. 3-12 Connections of coils in (a) a low range and (b) a high range dynamometer ammeters.

In the first case the fixed coils and the moving coil carry the same current, hence Eq. 3.16 is applicable. As spring control is being used and in practice dM/dθ is made constant,

$$S\theta = K'I^2$$
$$\theta = K_1 I^2$$

where S is the spring constant and

$$K_1 = \frac{K'}{S} = \frac{dM/d\theta}{S}$$

In the high range ammeters, as shown in Fig. 3-12(b), the fixed coil current is different from the moving coil and to make and phase angle between two currents ϕ=0, the time constants, L/R, of the two parallel paths are made equal. As both the currents are proportional to I, the current to be measured, the ratio of two currents must be constant, i.e. I_1/I_2=K'''. Hence,

$$T_D = K''I^2 \frac{dM}{d\theta} = K'''I^2 \quad \left(K''' = K''\frac{dM}{d\theta}\right) \tag{3-20}$$

$$\theta = K_2 I^2$$

where S is the spring constant and

$$K_2 = \frac{K'''}{S}.$$

Connection of voltmeter

As said earlier, the dynamometer voltmeter are most accurate form of voltmeter for measuring a.c voltage ranging from 50 to 500 V at power frequency. The fixed and moving coils are connected in series with a high non-inductive resistance as shown in Fig. 3-13. Since the current through the moving coil doesn't exceed 75 mA, ordinary control springs are used to lead the current.

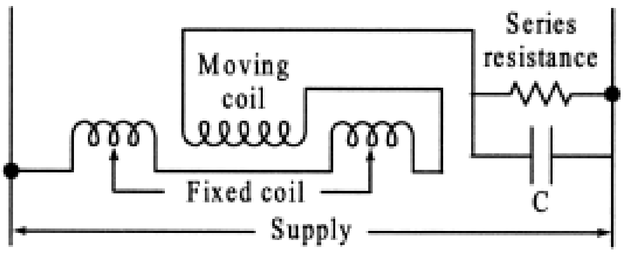


Fig. 3-13 Dynamometer Voltmeter

Since, the fixed and the moving coils are connected in series, hence Eq. 3.16 is applicable. Hence,

$$T_D = I^2 \frac{dM}{d\theta}$$

If the impedance of the circuit shown in Fig. 3-13 is z then

$$T_D = \frac{V^2}{Z^2} \frac{dM}{d\theta}$$

As spring control is being used and in practice *dM/dθ* is made constant,

$$S\theta = \frac{V^2}{Z^2} \frac{dM}{d\theta} \qquad (3\text{-}21)$$

$$\theta = K_3 V^2 \qquad (3\text{-}22)$$

where S is the spring constant and $K_3 = \frac{dM/d\theta}{SZ^2}$

Errors in dynamometer meters

Dynamometer meter have air cored coils and do not have permanent magnets. So, this type of meters is free from hysteresis errors and error due to weakening of the magnets. Other errors occurring in this type of meters are listed as follows:

1- Frictional error
2- Temperature error
3- Error due to stray magnetic fields
4- Frequency error

Frictional error

As mentioned earlier, the number of turns of the moving coil is more; its weight is higher for the same torque than that in permanent magnet meter. This increases the weight/torque ratio. So, the frictional error is high. To limit the frictional error the moving coil is wound with a very fine wire.

Temperature error

Like other instruments, the sources of temperature error are the self-heating and change in external temperature. As operating power required in dynamometer is large, self-heating is applicable and the error due to it maybe large. The change of temperature changes the spring constant and the coil resistances. In voltmeters high series resistance with low temperature coefficient takes care of the change of the coil resistance. In ammeters, the effect of change in resistance may be eliminated by matching the temperature coefficients of the shunt and the coil, but it is not so easy.

Error due to stray magnetic fields

The operation of this type meters is very sensitive to the stray magnetic fields as its operating field is small. The error can be minimized by proper shielding.

Frequency error

The change of frequency causes the following errors:

- Reactance error
- Eddy current error

Reactance error: the change in frequency changes the value of reactance, in turn changes the impedance of the circuit. As shown in Eq. (3-21), the reading of the voltmeter is dependent upon the impedance, so the change of impedance changes the reading of the voltmeter. The error may be minimized by using very high series resistance. A more effective method uses a capacitance in parallel with the series resistance as shown in Fig. 3-13.

The impedance of the circuit of Fig. 3-13, neglecting coil resistance,

$$Z = \frac{R(1 - j\omega CR)}{1 + \omega^2 C^2 R^2} + j\omega L$$

It may be assumed that $\omega^2 C^2 R^2 \ll 1$. Hence,

$$Z = R + j\,\omega(L - CR^2)$$

For the impedance to be independent of frequency,

$$C = \frac{L}{R^2} \qquad (3\text{-}23)$$

As said earlier, to make the deflecting torque constant for a current to be measured and independent of phase angle between the two circuit currents, i.e. ϕ=0, the ratio of two currents must be constant. The ratio of current must also be independent of frequency to eliminate frequency error in ammeters of Fig. 3-12. This can also be achieved by making the time constants of the two circuits same.

Eddy current error: Eddy current is induced in the metal parts of the meter as a result of coupling between the moving coil and he metal parts. The effect of the eddy current is to produce a torque causing an additional error. To reduce this error, the metal parts should be reduced to a minimum and the metal used should be of high resistance.

Ammeters

Fig. 3.14(a) shows that fixed coils and moving coil of a dynamometer instrument connected in series and assumed the current through moving coil does not exceed a certain the upper limit depending on its construction.

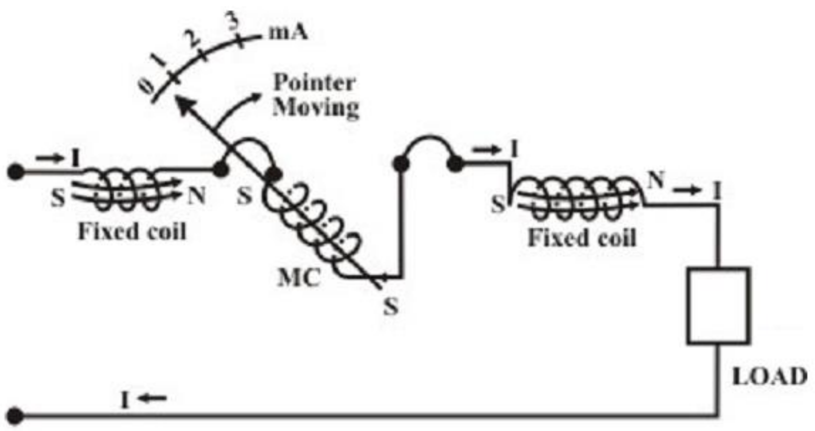


Fig. 3.14a Torque produces positive deflection (clockwise)

The flux direction through the fixed and movable coils due to current is shown in Fig. 3.14 (a). it can be noted that the –pole of the moving coil flux is reflected from the adjacent –pole of the fixed coil and on the other side adjacent S –poles are also repelled each other. This results the pointer to move clockwise direction from ‘zero position’ to a steady position depending upon the magnitude of current flowing through the coils. Fig. 3.14(b) illustrate the effect of reversing the direction of the current through the coils and shows that the deflecting torque produces movement of the pointer in the same direction. This means that the dynamometer instrument suitable for both dc and ac measurements of current and voltage. The dynamic instrument when uses as a voltmeter, the fixed coils wounded with thin wire are connected in series with the moving coil and a non-inductive resistance. For ammeter application the fixed coils are connected in parallel with the moving coil, and in parallel with a shunt, as required.

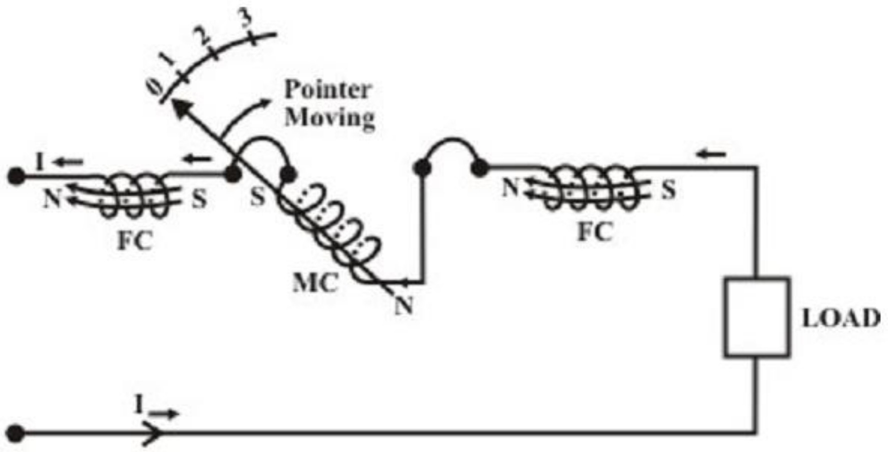


Fig. 3.14(b) Current flowing right to left produces positive deflection.

Remarks: The scale of the instrument can be conveniently calibrated on dc and then used to measure ac.

3.2.2 Ranges of Ammeters and Voltmeters

Ammeters

A given size of instruments requires a definite number of ampere-turns to be supplied by the fixed and moving coils to obtain a full–scale deflection. Ammeter ranges are altered by changing the number of turns and size of conductor in the fixed and moving coils. A double range instrument may easily be obtained by connecting different coil sections either in series or in parallel. The internal connections are shown in Fig. 3.15 (for higher range, fixed coils are in parallel; for lower range, fixed coils are in series). The maximum range for which ammeters are usually constructed is dependent on its application. For ammeter use in which only fraction of rated current (say 200 ma) is carried by the moving coil to alter its range by changing the mode of connection of the fixed coils.

Voltmeters: With voltmeters the ranges is altered by changing the number of turns in the coils and the value of series resistances, but the range of a given instrument may be increased by connecting additional resistances in series with it. For example, the range of a given voltmeter may be doubled while connecting in series with it a non–inductive resistance equal in value to the original resistance of the instrument.

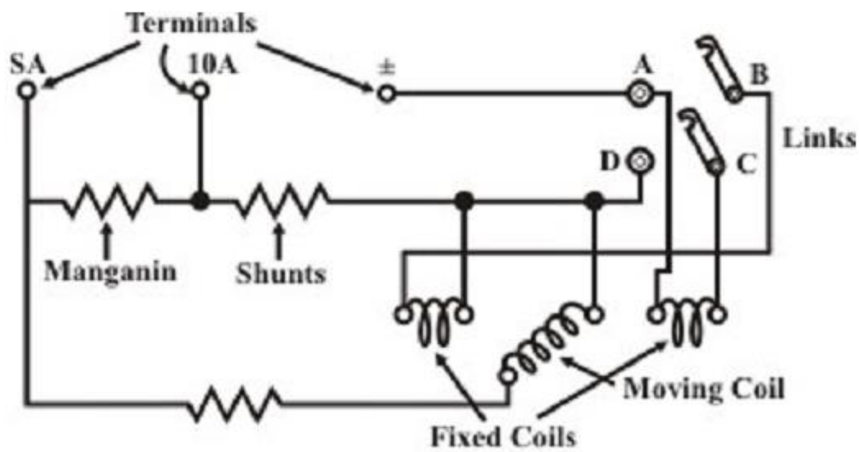


Fig. 3.15 Connection of double-range electro-dynamic ammeter.

3.8 Connections for ammeter, voltmeter and wattmeter

Ammeter

When ammeters for ranges above about 250 , the moving coil cannot be connected in series with the fixed coil (note the control spring is unsuitable for currents above about 250). Therefore, the moving coil must be connected in parallel with the fixed coils as shown in Fig 3.16.

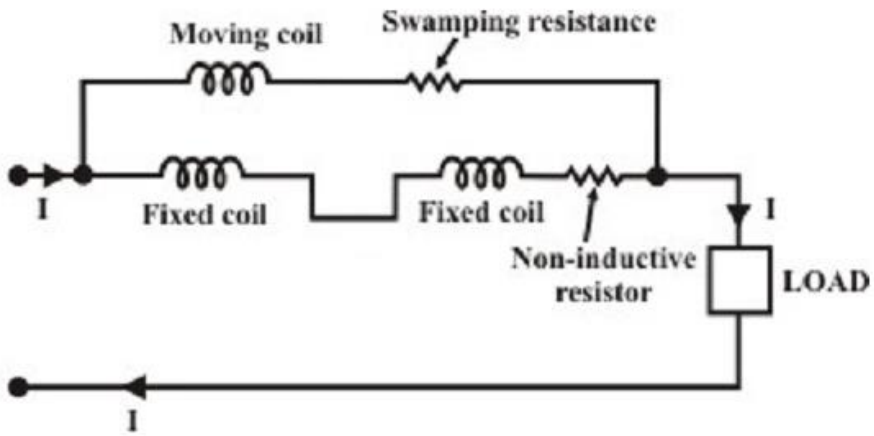


Fig. 3.16 Electro dynamic Ammeter (above 250mA) conncetion

Here the moving coil current is kept within 200 mA and the rest of current is passed through the fixed coil. Moving coil carries a small fraction of measured current through the moving coil.
For extreme accuracy the connection shown in Fig. 3.16 must fulfill the following conditions.

- The resistance/reactance ratio must have the same value (i.e time constant of moving coil= time constant of fixed coil) for each branch.
- The percentage. change of resistance with temperature must be the same for the two branches

The connection for use as a voltmeter is shown in Fig. 3.17, in which fixed and moving coils are connected in series with a high series resistance having "zero resistivity coefficients".
This combination is connected across the voltage source or across the load terminals whose voltage is to be measured. The deflecting torque is given by

$$T_d = i_f\, i_m \frac{dM}{d\theta} = \frac{V}{Z}\frac{V}{Z}\frac{dM}{d\theta} \qquad (3\text{-}30)$$

where Z is the magnitude of total impedance of the voltmeter circuit. At steady state condition of deflection

$$T_d = T_c$$

$$k_s\theta = \frac{V^2}{Z^2}\,\frac{dM}{d\theta}$$

$$\theta = \frac{V^2}{Z^2 k_s}\,\frac{dM}{d\theta} \qquad (43.10)$$

This implies that deflecting torque is directly proportional to V^2 if $\frac{dM}{d\theta}$ is kept nearly constant.

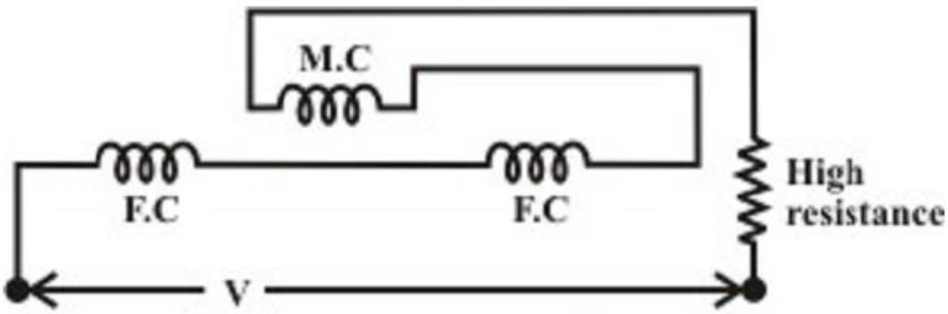


Fig. 3.17 Electro-dynamic voltmeter connection

The fixed coil is connected in series with the load to have the same load current. A typical connection of an electrodynamometer for use as a wattmeter is shown in Fig. 3.18.

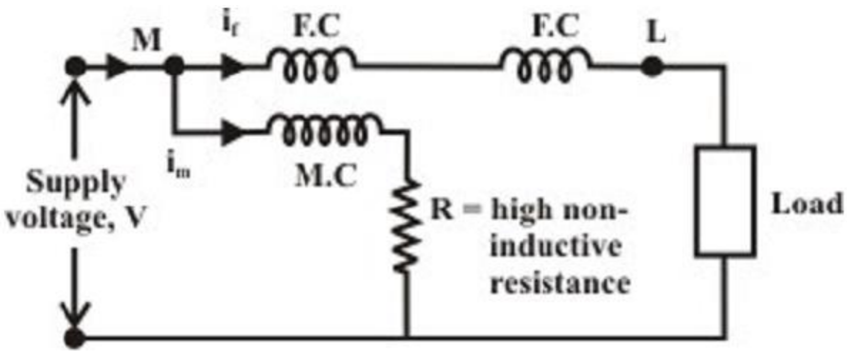


Fig. 3.18 Wattmeter connection

This is possible if θ varies from 45° to 135° over the range of instrument scale.

Remarks: Electro-dynamic meter's use is much more common for ac voltmeters than for ac ammeters because of practical limitation on current through the moving coil. Electroynamic ammeter needs to read r.m.s values of alternating current accurately irrespective of signal waveform or distortion of signal waveform.

Wattmeter: Perhaps the most important use of the electrodynamometer is for the wattmeter.

The mechanism of electro dynamic wattmeter closely resembles that of an electro-dynamic ammeter, but the moving coil of wattmeter is connected in series with a high non–inductive resistance. It provides with separate terminals to connect across the load terminals.

3.8.8. Electrodynamic or Dynamometer Type Moving Coil Instruments

3.8.8.1. Introduction

In this type of instrument, the *operating field is produced by another fixed coil* and *not by a permanent magnet*.

- These instruments are capable of service as *transfer instruments* (A '***transfer instrument***' *is one that may be calibrated with a D.C. source and then used without modification to measure A.C.*). Indeed, their principal use as ammeters and voltmetes in laboratory and measurement work is for the transfer calibration of working instruments and *as standards for calibration of other instruments a their accuracy is very high.*
- These instruments are employed as *A.C. voltmeters and ammeters both in the range of power frequencies and lower part of the audio frequency range.* They are used as *watt-meters, var-meters and with some modification as power factor meters and frequency meters.*

3.8.8.2. Construction and working

Fig. 3.44 shows a dynamometer type instrument :

The *main components* of the instrument are :

(*i*) Field system (Fixed coil).

(*ii*) Moving system (Moving coil).

(*iii*) Control system (Hair springs).

(*iv*) Damping system (Air friction damping).

(*v*) Shielding.

(*vi*) Cases and scales.

— These instruments essentially consist of *fine wire moving coil* placed in the magnetic field produced by *another fixed coil* when carrying currents.

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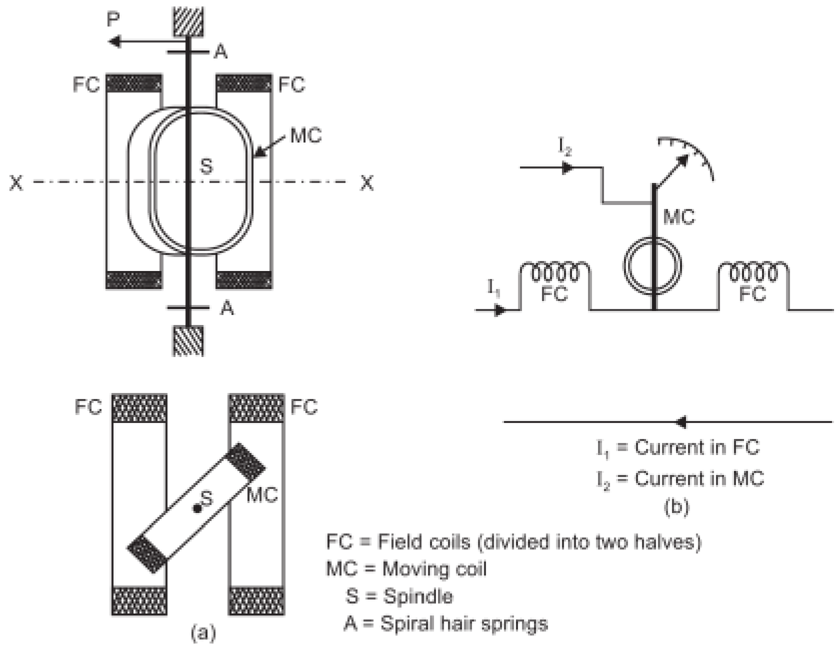

Fig. 3.44. Electrodynamic or dynamometer type instrument.

— The coils are usually *air cored* to *avoid hysteresis, eddy currents and other errors* when the instrument is used on A.C. The *fixed coil FC* is divided into *two halves* placed close together and *parallel* to each other in order to provide a *fairly uniform field*, within the range of the movement of the moving coil.

— The diagram in Fig. 3.44(*a*) shows a sectional elevation through fixed coil *FC* and the lower diagram represents a sectional plan on *XX*. The moving coil *MC* is carried by a spindle S and the *controlling torque is exerted by spiral hair spring A*, which may also serve to lead the current into and out of MC.

— In these instruments *air friction damping is used* and may be either piston type or vane type. Eddy current damping *cannot* be used as introduction of a permanent magnet for the purpose would *distort* the working magnetic field of the instrument.

— The complete assembly is surrounded by a *laminated steel shield to protect the instrument from external magnetic field* which may affect the operation of the instrument.

— Laboratory standard instruments are usually contained in *highly polished wooden* cases. The case is supported by adjustable levelling screws. A spirit level is also provided to ensure proper levelling. The *scales* are hand drawn, using machine sub-dividing equipment.

Deflecting torque. The deflecting torque is due to *interaction of the magnetic fields produced by currents in the fixed and moving coils.*

— Fig. 3.45(*a*) shows the magnetic field due to current flowing through *FC* (I_1) in the direction indicated by the dots and cross.

— Fig. 3.45(*b*) shows the magnetic field due to current (I_2) in *MC*.

— Fig. 3.45(*c*) shows the combined effect of the above magnetic fields. By combining these magnetic fields it will be seen that when currents (I_1 and I_2) flow simultaneously through *FC* and *MC*, the resultant magnetic field is *distorted* and effect is to exert a clockwise torque on *MC*.

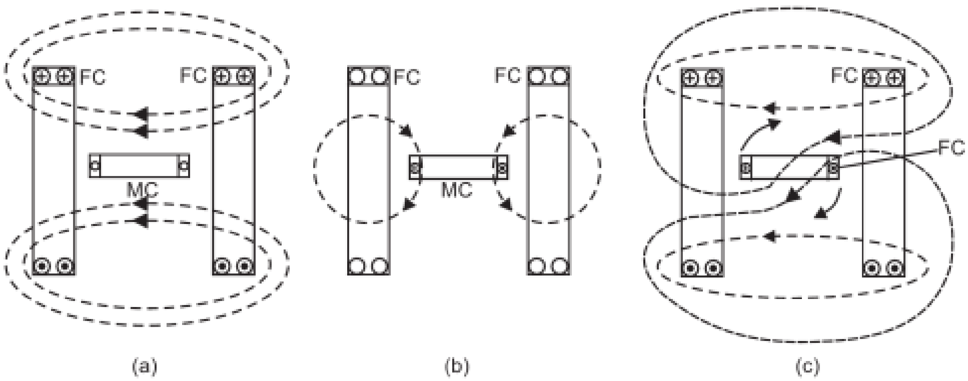

Fig. 3.45. Magnetic fields due to fixed and moving coils.

Since *MC* is carrying current (I_2) at right angles to the magnetic field produced by *FC*, *deflecting torque,*

$$T_d \propto I_1 \times I_2$$

or, $T_d = KI_1I_2$ where, K is a constant.

Since the instrument is spring-controlled, the *restoring or control torque* (T_c) is proportional to the angular deflection θ.

$$T_c \propto \theta \quad \text{or} \quad T_c = K'\theta$$

The two torques (T_d and T_c) are equal and opposite in the final deflection position.

i.e., $T_d = T_c$

or, $KI_1I_2 = K'\theta$

or, $\theta \propto I_1I_2$

or, $\theta \propto I^2$ since $I_1 \propto I$ and $I_m \propto I$

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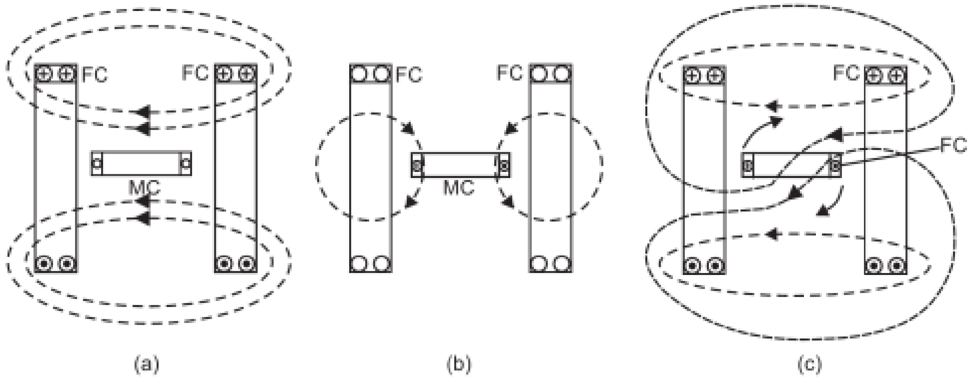

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Expression for torque in terms of mutual inductance M between fixed and moving coils :

The total energy stored in the magnetic field of the fixed coil,

$$E = \frac{1}{2}L_1I_1^2 + \frac{1}{2}L_2I_2^2 + I_1I_2M \quad \text{...(3.89)}$$

where L_1 and L_2 are the self inductances of the fixed coil and moving coil respectively.

The interaction of these currents give rise to a deflecting torque, T_d that sets the moving coil in a position for which the energy of the coil magnetic field attains maximum value.

Deflecting torque, $$T_d = \frac{dE}{d\theta} = \frac{1}{2}\cdot\frac{dL_1}{d\theta}\cdot I_1^2 + \frac{1}{2}\frac{dL_2}{d\theta}I_2^2 + I_1I_2\frac{dM}{d\theta}$$

where $d\theta$ is the increase in angular deflection at which the field energy increases by dE.

Since coil inductances L_1 and L_2 are constant, therefore, dL_1 and dL_2 each is equal to zero and T_d is given by :

$$T_d = I_1I_2\frac{dM}{d\theta} \quad \text{...(3.90)}$$

For direct current :

$$T_d = I_1 I_2 \frac{dM}{d\theta}$$

where I_1 and I_2 are the currents in fixed coil and moving coil respectively.

The above expression shows that the deflecting torque depends in general both on currents I_1 and I_2 and position of the moving coil with respect to the fixed one. The dependence of the rate of change of mutual inductance on the angular deflection of the moving coil $\frac{dM}{d\theta} = f(\theta)$ is determined by the *shape of coils and their relative positions.*

Deflecting torque (T_d) rotates the moving coil through an angle θ, at which restoring torque (T_c) of strings is equal to T_d.

$\therefore$ $T_d = T_c = C\theta$, where C is spring restoring constant (Nm/rad.)

or, $$I_1 I_2 \frac{dM}{d\theta} = C\theta$$

or, Deflection, $$\theta = \frac{I_1 I_2}{C} \cdot \frac{dM}{d\theta} \qquad ...(3.91)$$

For alternating current :

When the coils carry alternating currents i_1 and i_2 the instantaneous deflecting torque,

$$(T_d)_i = i_1 i_2 \frac{dM}{d\theta} \qquad ...(3.92)$$

The average deflecting torque over a complete cycle,

$$(T_d)_{av.} = \frac{1}{T}\int_0^T T_{di}\, dt$$

or, $$(T_d)_{av.} = \frac{dM}{d\theta}\frac{1}{T}\int_0^T i_1 i_2\, dt \qquad ...(3.93)$$

or, $$(T_d)_{av.} = \frac{dM}{d\theta}\frac{1}{T}\int_0^T i_1 i_2 \, dt \qquad ...(3.93)$$

where, T = Time period for one complete cycle.

If currents i_1 and i_2 are *sinusoidal and are displaced by a phase angle* ϕ, *i.e.*,

$$i_1 = I_{max.1} \sin \omega t, \quad \text{and} \quad i_2 = I_{max.2} \sin(\omega t - \phi),$$

then, the *average deflecting torque*,

$$(T_d)_{av.} = \frac{dM}{d\theta}\cdot\frac{1}{T}\int_0^T i_1 i_2 \, dt$$

$$= \frac{dM}{d\theta}\cdot\frac{1}{2\pi}\int_0^{2\pi} I_{max.1} \sin \omega t \, . \, I_{max.2} \sin(\omega t - \phi) \, d(\omega t)$$

or, $$(T_d)_{av.} = \frac{I_{max.1} \, . \, I_{max.2}}{2} \cos\phi \frac{dM}{d\theta} = I_1 I_2 \cos\phi \frac{dM}{d\theta} \qquad ...(3.94)$$

where I_1 and I_2 are r.m.s. values of current flowing in the coils.

At steady deflection state,

$$T_d = T_c$$

or, $$(T_d)_{av.} = \frac{dM}{d\theta}\frac{1}{T}\int_0^T i_1 i_2 \, dt \qquad ...(3.93)$$

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where I_1 and I_2 are r.m.s. values of current flowing in the coils.

At steady deflection state,

$$T_d = T_c$$

or, $\quad I_1 I_2 \cos\phi \dfrac{dM}{d\theta} = C\theta$

$$\therefore \quad \text{Deflection, } \theta = \frac{I_1 I_2}{C} \cos\phi \cdot \frac{dM}{d\theta} \qquad ...(3.95)$$

From the above discussion it is obvious that for a *sinusoidal alternating currents the deflecting torque and the deflection are determined by the product of r.m.s. values of coil currents and cosine of the phase angle between them.* It is clear that the *dynamometer type of instruments can be used on both D.C. and A.C.*

Use of the instrument as an ammeter. When the instrument is used as an ammeter then same current passes through both moving coil (*MC*) and fixed coils (*FC*) as shown in Fig. 3.46. In this case, $I_1 = I_2 = I$, hence $\theta \propto I^2$ or $I \propto \sqrt{\theta}$. The connections of Fig. 3.46 are used when *small currents* are to be measured.

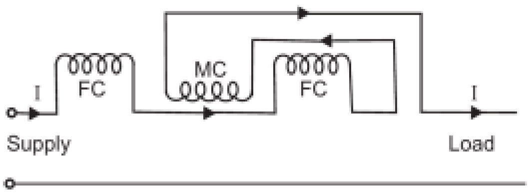

Fig. 3.46. Measurement of small currents.

In the case of heavy currents, a shunt is used to limit current through the moving coil as shown in Fig. 3.47.

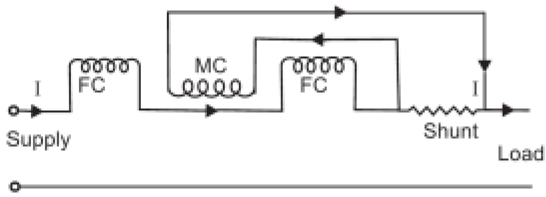

Fig. 3.47. Measurement of heavy currents.

Use of the instrument as voltmeter. When the instrument is used as a voltmeter, the fixed and moving coils are used in series along with high resistance as shown in Fig. 3.48.

Here again, $I_1 = I_2 = I$,

where, $\quad I = \dfrac{V}{R}$ ......in D.C. circuits

and, $\quad I = \dfrac{V}{Z}$ ......in A.C. circuits

$\therefore \quad \theta \propto V \times V$ or $\theta \propto V^2$

or, $\quad V = \sqrt{\theta}$

- Thus, whether the instrument is used as an ammeter or voltmeter, its scale is *uneven* through the whole of its range and is cramped or crowded near the zero in particular.

Note: When the dynamometer instrument is used to measure an alternating current or voltage, the moving coil due to its inertia takes up a position where the average deflecting torque over one cycle is balanced by the restoring torque of the spiral springs. For that position, the deflecting torque is proportional to the mean value of the square of current or voltage, and the instrument scale can therefore be *calibrated to read the r.m.s. value.*

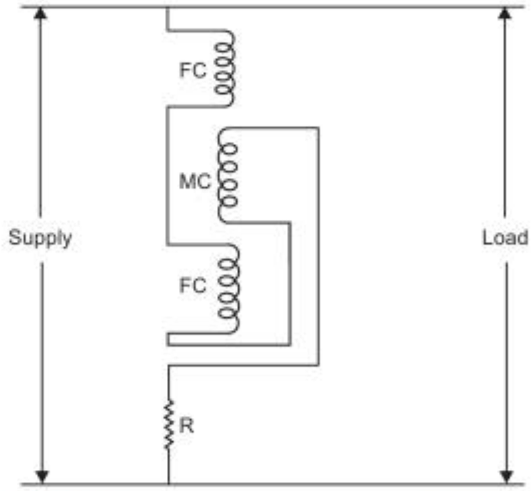

Fig. 3.48. Use of the instrument as a voltmeter.

Ranges :

Ammeter. (*i*) With fixed and moving coils in series .... 0/0.01 A – 0/0.05 A

(*ii*) With moving coil shunted or parallel connections .... upto 0/30 A.

Voltmeters. Upto 0–750 volts.

3.8.8.3. Errors in dynamometer type instruments

Following errors usually occur in these instruments :

(*i*) Frictional error;

(*ii*) Temperature error;

(*iii*) Error due to stray magnetic fields;

(*iv*) Frequency error;

(*v*) Error due to eddy currents.

1. ***Frictional error :*** In these instruments frictional error is high. So, in order to have high accuracy of measurement this error needs to be minimised to the lowest possible, this is achieved as follows :

(*i*) By a reasonable reduction in weight;

(*ii*) By proper selection of materials for bearings and spindle, and adequate polishing of these parts.

However, these measures increase the cost of the instrument and make it more sensitive to overloads and mechanical impacts.

2. ***Temperature error :***

- High currents are carried by coils which produce heat. *Self-heating of coils produces errors.*
- High accuracy instruments contain *temperature compensating resistors* which tend to *neutralise* the effects of changes in temperature.

3. ***Errors due to stray magnetic fields :*** In order to avoid the influence of stray fields a good amount of *screening is necessary.*

- In precision instruments the influence of external magnetic fields can be minimised by using *"Astatic system"*.

— *"Astatic"* dynamometer type instruments are constructed with two similar sets of fixed and moving coils mounted on the same shaft. The two pairs of fixed coils are so connected that their *magnetic fields are in opposition. The currents in the moving coils are also opposed* and, in consequence, the deflection torques applied to the moving coils act in the same direction. Since the two fields are in opposition and, therefore, any uniform external field reduces the field of one coil and increases by an equal amount the field of the other coil, the deflecting torque produced by one coil is increased while that produced by the other coil is reduced by an equal amount and, thus *net torque on account of external magnetic field is zero.*

4. ***Frequency error :*** This error in these instruments is largely due to variation of self-reactance of coils, with frequency.

- In order to reduce frequency error in dynamometer ammeters, the ratio of currents in fixed and moving coils *i.e.* $\frac{I_1}{I_2}$ should be independent of frequency. This requires that *time constant* $\left(\frac{L}{R}\right)$ *of two circuits should be same.*
- For reducing frequency errors in voltmeters the *coil winding is made a very small part of the circuit.* This makes inductive reactance ($X_L = 2\pi fL$), a *small fraction* of total impedance, and therefore $Z = \sqrt{R^2 + (2\pi fL)^2} = R$. This means frequency does not affect the calibration of the instrument. In fact, dynamometer voltmeters may be used, in general, within their guaranteed accuracy from D.C. to about 125 Hz.

5. ***Error due to eddy currents :***

- The eddy currents are induced in metal parts of the instrument and develop a torque because of coupling between the moving coil and the neighbouring metal part.
- This error can be *reduced* by keeping the metal in the supports of the coil and other structural parts to the *minimum possible* and selecting the metal, to be used, of *high resistivity.*

3.8.8.4. Advantages and disadvantages

Advantages :

1. Can be used on both D.C. as well as A.C. systems (as the instrument has square law response).
2. They are free from hysteresis and eddy current errors, because of absence of iron in the operating parts of the instrument.
3. It is possible to construct ammeters upto 10 A and voltmeters upto 600 V with precision grade accuracy.
4. Very useful for accurate measurement of r.m.s. values of voltages irrespective of waveforms.
5. Owing to precision grade accuracy and same calibration for D.C. and A.C. measurements, these instruments are used as *transfer and calibration instruments.*

Disadvantages :

1. Since torque/weight ratio is small, such instruments have *low sensitivity.*
2. The *scale is not uniform* because $\theta \propto \sqrt{I}$.
3. Cost of these instruments is *higher* in comparison to those of moving-iron instruments. So, these are *only used as voltmeters and ammeters for precision measurements.*
4. *Higher frictional losses.*
5. These instruments are sensitive to overloads and mechanical impacts. Therefore, they *must be handled with care.*
6. In these instruments the operating current is large and consequently they have a *higher power consumption than PMMC instruments.*

Example 3.52 The mutual inductance of a 25 A electrodynamometer ammeter changes uniformly at a rate of 0.0035 μH/degree. The torsion constant of the controlling spring is 10-6 N.m/degree. Determine the angular deflection for full scale.

Solution. *Given* : $I_1 = I_2 = I = 25$ A; $\frac{dM}{d\theta} = 0.0035$ μH/degree; $C = 10^{-6}$ Nm/degree

Angular deflection for full scale, θ :

Now, $$T_d = I^2 \frac{dM}{d\theta}$$ [Eqn. (3.90)]

... instrument being ammeter ($I_1 = I_2$)

Since spring constant is given Nm/degree, therefore, we must express $\frac{dM}{d\theta}$ in H/rad. if deflection is to be found in degrees.

$$\therefore \quad \frac{dM}{d\theta} = \frac{0.0035 \times 10^{-6}}{(\pi/180)} = 0.2 \times 10^{-6} \text{ Nm/rad.}$$

Full scale deflection torque,

$$T_d = T_c = C\theta = 10^{-6}\theta \text{ Nm}$$

Substituting this value of T_d in the above eqn., we get :

$$10^{-6}\theta = (25)^2 \times 0.2 \times 10^{-6}$$

or $$\theta = \mathbf{125°} \quad \textbf{(Ans.)}$$

Example 3.53. *A 10 A electrodynamometer is controlled by a spring having a torsion constant of 0.1 × 10^{-6} Nm/degree. The full scale deflection is 110°. Determine the inductance of the instrument when measuring a current of 10 A. The mutual inductance at 0° deflection is 2 μH and the change in mutual inductance is linear as the deflection.* **(Nagpur University)**

Solution. *Given* : $C = 0.1 \times 10^{-6}$ Nm/degree; $\theta = 110°$, $I = 10$ A; M at 0° deflection = 10A

Inductance of the instrument :

Full scale deflecting torque, $T_d = T_c = C\theta = 0.1 \times 10^{-6} \times 110 = 11.1 \times 10^{-6}$ Nm

Also, $$T_d = I^2 \frac{dM}{d\theta}$$

$$\therefore \quad \frac{dM}{d\theta} = \frac{T_d}{I^2} = \frac{11.1 \times 10^{-6}}{(10)^2} = 0.111 \ \mu\text{H/rad.}$$

Change in inductance, $$dM = 0.111 \times 10^{-6} \times 110 \times \frac{\pi}{180} = 0.213 \ \mu\text{H}$$

∴ *Total inductance of the instrument* = Initial inductance + change in inductance

= 2 + 0.213 = **2.213 μH** **(Ans.)**

Example 3.54. *The spring constant of a 10 A dynamometer wattmeter is 10.5 × 10^{-6} Nm/radian. The variation of inductance with angular position of moving system is practically linear over the operating range, the rate of change being 0.078 mH/radian. If the full-scale deflection of the instrument is 83 degrees, calculate the current required in the voltage coil at full scale on D.C. circuit.* **(Nagpur University)**

Solution. *Given* : $C = 10.5 \times 10^{-6}$ Nm/rad.; $10.5 \times 10^{-6} \times \frac{\pi}{180}$ Nm/degree; $\frac{dM}{d\theta} = 0.078$ mH/rad.; $\theta = 83°$.

Current required in voltage coil, I_2 :

$$T_d = T_c = C\theta = \left(10.5 \times 10^{-6} \times \frac{\pi}{180}\right) \times 83 = 15.21 \times 10^{-6} \text{ Nm}$$

Also, $$T_d = I_1 I_2 \frac{dM}{d\theta} \quad \text{[Eqn. (3.90)]}$$

$\therefore$ $$15.21 \times 10^{-6} = 10 \times I_2 \times 0.078$$

or, $$I_2 = 19.5 \times 10^{-6} \text{ A} \quad \text{or} \quad \mathbf{19.5\ \mu A} \quad \textbf{(Ans.)}$$

Example 3.55. *In a certain dynamometer ammeter the mutual inductance M varies with deflection θ (expressed in degees) as :*

$$M = -8 \cos(\theta + 30°) \text{ mH}$$

Find the deflecting torque produced by a direct current of 60 mA corresponding to a deflection of 60°.

Solution. *Given* : $M = -8 \cos(\theta + 30°)$ mH ; $I = 60$ mA or 0.06 A; $\theta = 60°$

Deflecting torque, T_d :

Rate of change of mutual inductance with deflection,

$$\frac{dM}{d\theta} = \frac{d}{d\theta}[-8\cos(\theta + 30°)] = 8 \sin(\theta + 30°) \text{ mH}$$

$\frac{dM}{d\theta}$ at a deflection of 60° *i.e.*, $\left(\frac{dM}{d\theta}\right)_{\theta = 60°} = 8 \sin(60° + 30°) \text{ mH} = 8 \times 10^{-3}$ H/degree

$$\therefore \text{ Deflecting torque, } T_d = I^2 \frac{dM}{d\theta} = (0.06)^2 \times 8 \times 10^{-3} = \mathbf{28.8\ \mu Nm} \quad \textbf{(Ans.)}$$

Deflecting torque, T_d :

Rate of change of mutual inductance with deflection,

$$\frac{dM}{d\theta} = \frac{d}{d\theta}[-8\cos(\theta + 30°)] = 8 \sin(\theta + 30°) \text{ mH}$$

$\frac{dM}{d\theta}$ at a deflection of 60° *i.e.*, $\left(\frac{dM}{d\theta}\right)_{\theta=60°} = 8 \sin(60° + 30°) \text{ mH} = 8 \times 10^{-3}$ H/degree

$\therefore$ Deflecting torque, $T_d = I^2 \frac{dM}{d\theta} = (0.06)^2 \times 8 \times 10^{-3} =$ **28.8 μNm** **(Ans.)**

Example 3.56. *In a torsion-head type electrodynamometer a current of 36 A requires a deflection of 90° in the head to give balance.*

Determine :

(*i*) *The range of the instrument (maximum angle 360°)*

(*ii*) *For what current will the deflection be 180° ?*

(*iii*) *What will be the deflection for a current of 30 A ?*

Solution. In torsion-head electrodynamometer,

Torque, $T \propto (\text{current})^2$

Also, torque, $T \propto \theta$ (deflection)

$\therefore \quad \theta \propto I^2$

or, $\quad I \propto \sqrt{\theta}$

(*i*) Current corresponding to deflection of 360°,

$$I_2 = I_1 \times \sqrt{\frac{\theta_2}{\theta_1}} = 36 \times \sqrt{\frac{360}{90}} = \textbf{72 A} \textbf{ (Ans.)}$$

(*ii*) Current corresponding to deflection of 180°,

$$I_3 = I_1 \times \sqrt{\frac{\theta_2}{\theta_1}} = 36 \times \sqrt{\frac{180}{90}} = \textbf{50.9 A} \quad \textbf{(Ans.)}$$

(*iii*) Deflection corresponding to current 30 A,

$$\theta_4 = \theta_1 \times \left(\frac{I_4}{I_1}\right)^2 = 90 \times \left(\frac{30}{36}\right)^2 = \textbf{62.5°} \quad \textbf{(Ans.)}$$

Example 3.57. *A 50 V range spring controlled, electrodynamic voltmeter having a square law scale response takes 0.05 A on D.C. for full scale deflection of 90°. The control constant is 0.5 × 10^{-6} Nm/degree and the initial mutual inductance of the instrument is 0.25 H. Find the true potential difference across the instrument when it reads 50 V at 50 Hz.*

Solution. *Given* : $V = 50$ volts; $I = 0.05$ A at $\theta = 90°$; $C = 0.5 \times 10^{-6}$ Nm/degree; Initial inductance = 0.25 H; $f = 50$ Hz.

True p.d. across the instrument :

Full scale deflection controlling torque, $T_c = 0.5 \times 10^{-6} \times 90 = 45 \times 10^{-6}$ Nm

For steady deflection state, $T_d = T_c$

$$\therefore \quad I^2 \frac{dM}{d\theta} = 45 \times 10^{-6}$$

$$\text{or,} \quad \frac{dM}{d\theta} = \frac{45 \times 10^{-6}}{I^2} = \frac{48 \times 10^{-4}}{(0.05)^2} = 18 \text{ mH/rad.}$$

Total change in mutual inductance for deflection of 90° or $\frac{\pi}{2}$ rad.,

$$dM = 18 \times 10^{-3} \times \frac{\pi}{2} = 0.02827 \text{ H}$$

∴ Total mutual inductance (M) = Initial mutual inductance + change in mutual inductance

$$= 0.25 + 0.02827 \simeq 0.2783 \text{ H}$$

Reactance of the instrument at 50 Hz = $2\pi f M = 2\pi \times 50 \times 0.2783 = 87.43\ \Omega$

Resistance of the instrument, $R = \frac{V}{I} = \frac{500}{0.05} = 1000\ \Omega$

Impedance of the instrument at 50 Hz, $Z = \sqrt{(1000)^2 + (87.43)^2} = 1003.8\ \Omega$

∴ *True p.d. across the instrument* = $IZ = 0.05 \times 1003.8 =$ **50.19 V** **(Ans.)**

Example 3.58. *A 50 V range spring controlled electrodynamic voltmeter has an initial inductance of 0.25 H, the full scale deflection torque of 0.4 × 10^{-4} Nm and full scale deflection current of 50 mA.*

Determine the difference between D.C. and 50 Hz A.C. readings at (i) 50 V and (ii) 25 V if the voltmeter inductance increases uniformly over the full scale of 90°. **(Delhi University)**

Solution. *Given* : Initial inductance = 0.25 H; $T_d = 0.4 \times 10^{-4}$ Nm; I = 50 mA or 0.05A

Now, $$T_d = I^2 \frac{dM}{d\theta}$$

or, $$0.4 \times 10^{-4} = (0.05)^2 \times \frac{dM}{d\theta}$$

or, $$\frac{dM}{d\theta} = \frac{0.4 \times 10^{-4}}{(0.05)^2} = 0.016 \text{ H/rad.}$$

Total change in inductance for full scale deflection,

$$dM = 0.016 \times \left(90 \times \frac{\pi}{180}\right) = 0.0251 \text{ H}$$

$\therefore$ Total mutual inductance, $M = 0.25 + 0.251 = 0.2751$ H

At 50 V :

The resistance of voltmeter, $R = \frac{V}{I} = \frac{50}{0.05} = 1000\ \Omega$

The impedance while measuring the voltage of 50 V at 50 Hz A.C.,

$$Z = \sqrt{(1000)^2 + (2\pi \times 50 \times 0.2751)^2} = 1003.7\ \Omega$$

$$\therefore \quad \text{Voltmeter reading} = \frac{50}{1003.7} \times 1000 = 49.8 \text{ V}$$

Hence, *difference in readings* = 50 − 49.8 = **0.2 V** **(Ans.)**

At 25 V :

Current through instrument, $I' = \frac{25}{1000} = 0.025$ A

$$\text{Deflection, } \theta' = \theta \times \frac{(I')^2}{(I)^2}, \quad \text{since } \frac{dM}{d\theta} \text{ is constant}$$

$$= \frac{\pi}{2} \times \left(\frac{0.025}{0.05}\right)^2 = 0.3927 \text{ rad.}$$

$$\text{Mutual inductance at 25 V} = \text{Initial inductance} + \frac{dM}{d\theta} \times \theta'$$

$$= 0.25 + 0.016 \times 0.3927 = 0.2563 \text{ H}$$

$$\text{Impedance at 25 V,} \quad Z' = \sqrt{(1000)^2 + (2\pi \times 50 \times 0.2563)^2} = 1003.24\ \Omega$$

$$\therefore \quad \text{Voltmeter reading} = \frac{25}{1003.24} \times 1000 = 24.92 \text{ V}$$

Hence, difference in reading = 25 − 24.92 = **0.08 V** **(Ans.)**

References

Fundamentals of Electrical Engineering 2nd Ed., Rajendra Prasad - 2009

Questions

Q-27. What components in an electrodynamometer-type meter movement produce the magnetic field?

Q-28. What is the limiting factor as to the amount of current an electrodynamometer meter movement can handle?

Q-29. What is the primary advantage of the electrodynamometer-type meter over the D'Arsonval-type meter?

Problems

1. Derive an expression for the torque on a dynamometer ammeter in terms of the currents in the coils and the rate of change of mutual inductance with deflection. [10]

2. A dynamometer wattmeter is connected with the voltage coil on the supply side of the current coil. Derive an expression for a correction factor to allow for the inductance of voltage coil. Calculate the correction factor of the wattmeter if the phase angle of the voltage coil is 1° and the power factor of the load is (i) 0.8 lagging(ii) 0.8 leading. [8]

[Answer: (i) 0.987, (ii) 1.013.]

3. An electro dynamic wattmeter is used to measure the power consumed by the load. The load voltage is 250 V and the load current is 10A at a lagging power factor of 0.5. The wattmeter voltage circuit has a resistance of 2000 Ω and inductance 40 mH. The voltage coil (moving coil) is connected directly across the load. Estimate the percentage error in the wattmeter reading. (Assume the supply frequency = 50 HZ)

(Answer: 0.2% high)

4. A dynamometer ammeter is arranged so that 1/100th of the total current passes through the moving coil and the remainder through the fixed coil. The mutual inductances between the two coils varies with the angle of displacement of the moving coil from its zero position as indicated below:

θ	0	15°	30°	60°	90°	105°	120°
M(μH)	-336	-275	-192	0	192	275	336

If a torque of is required to give a full-scale deflection of 120°, calculate the current at half and full-scale deflection.

(Answer: 1.187A, 2.46A)

5. A voltmeter has a range of 120 volts and a resistance of 1550 Ω at 20°C, of which 77 Ω is due to the resistances of fixed and moving coils and the remainder 1473 Ω for non-inductive resistance connected to the moving coil. The inductances of the instrument (measured at 50 *Hz*) for number of position of the moving system are given in Table. When voltmeter used on a 100 volt dc circuit and 50 *Hz*, 100 volt a.c. circuit.

Applied voltage	0	40	60	80	100	120
Angular deflection	0	7	13.8	24	37.1	54
Inductance (mH)	70.1	72.5	74.8	78.3	82.8	88.6

The inductance of the fixed coil is 74.5 mH and moving coil is 2.2 mH.

Calculate (i) Mutual inductance between fixed and moving coils against applied voltage using above data
(ii) Plot mutual inductances against angular deflection and comments on $dMd\theta$.
(iii) Power loss in the instrument. [6+3+3]

Answer: (i)

Applied voltage	0	40	60	80	100	120
Mutual inductance (mH)	-3.3	-2.1	-0.95	0.8	3.05	5.95

(ii) $\frac{dM}{d\theta}$ is constant.
(iii) 6.45 watts.

6. An EDMM with a full scale deflection rating of 10 mA is to be used as a 1 mA ammeter. If the resistance of the moving coil is 40 Ω, what is the value of the shunt resistor Rsh?

7. Determine the reading obtained with a DC voltmeter at R_B when the switch is set at point A. Determine the reading at the same R_B using 1/2 wave and full wave rectifier AC meter respectively when the switch is set at point B. Given that I_{fs}=100 µA and set at 10 Vdc or rms range.

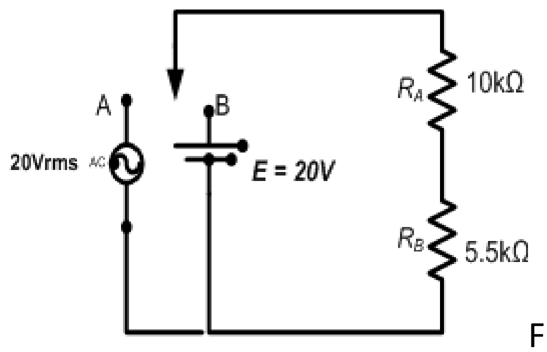

Fig. Problem 7